Lecture 28

$\boxed{\text{E.g.}}$

$$\det\begin{pmatrix} 2 & 3 & 7 \\ 0 & 0 & 0 \\ 1 & -3 & 2 \end{pmatrix} = \begin{vmatrix} 2 & 3 & 7 \\ 4 & 3 & 2 \\ 1 & -3 & 2 \end{vmatrix} \cdot 0 = 0$$

<u>Prop</u>: If $A = m \times n$ matrix has a row equal to $\vec{0}$, then $\det A = 0$.

$\boxed{\text{Eg}}$

$$\begin{vmatrix} 0 & 0 & 4 \\ 0 & 2 & 3 \\ 6 & 1 & -7 \end{vmatrix} \begin{matrix} \\ \\ -R_1 \end{matrix} = \begin{vmatrix} 0 & 0 & 4 \\ 0 & 2 & 3 \\ 6 & 1 & -11 \end{vmatrix} \begin{matrix} +R_3 \\ \\ \\ \end{matrix} = \begin{vmatrix} 6 & 1 & -7 \\ 0 & 2 & 3 \\ 6 & 1 & -11 \end{vmatrix} \begin{matrix} \\ \\ -R_1 \end{matrix} = \begin{vmatrix} 6 & 1 & -7 \\ 0 & 2 & 3 \\ 0 & 0 & -4 \end{vmatrix} \cdot \underbrace{(-1)}$$

This operation does change the determinant

$$= (-1)\begin{vmatrix} 6 & 1 & -7 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{vmatrix} = (-1)\cdot 6 \cdot 2 \cdot (4) \begin{vmatrix} 1 & 1/6 & -7/6 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1 \end{vmatrix} \cdots (-48)\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

<u>Prop</u>: If an $m \times n$ matrix B is obtained by swapping two rows of an $m \times n$ matrix A, then $\det(B) = -\det(A)$

<u>Prop</u>: If A is an $n \times n$ matrix and is upper triangular, then the $\det(A) = a_{11} \cdot a_{22} \cdot a_{nn}$ (product of diagonal entries)

$$\begin{bmatrix} a_{11} & \# & \# \\ 0 & a_{22} & \# \\ 0 & 0 & a_{nn} \end{bmatrix}$$

$\boxed{\text{Ex}}$

$$\begin{vmatrix} 2 & 3 & 7 \\ 0 & 9 & 3 \\ 1 & -3 & 2 \end{vmatrix} \leftarrow \text{this is } (R_1 - 2R_3) \text{ from this matrix: } \begin{pmatrix} 2 & 3 & 7 \\ 0 & 0 & 0 \\ 1 & -3 & 2 \end{pmatrix}$$

So both determinants $= 0$.

Theorem: The following are equivalent for $A \in M_{n \times n}$

1) $\det A = 0$
2) The rows of A are linearly dependent ☆
3) A is not row equivalent to I
4) The columns of A are linearly dependent
5) $\det A^T = 0$
6) A is not invertible

Prop: If A is an one by one matrix then $f(A) = a_{11}$ is a determinant function (f_n).

$\det[3] = 3$

Prop: The function $f: 2\times 2 \text{ matrix} \to \mathbb{R}$ given by $f\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$ is a determinant function.

$$\begin{vmatrix} 3 & 7 \\ -10 & 9 \end{vmatrix} = 27 - (-10)(7) = 97$$